# NEW RESULTS ON THE CEPHEID DISTANCE SCALE

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## ABSTRACT

We have applied an approximately Bayesian analysis to the calculation of Cepheid distances and radii using the surface brightness (Baade - Wesselink) method and a fully Bayesian analysis to the errors-in-variables portion of that problem. We demonstrate the use of these methods on the Galactic Cepheid distance scale. Both methods are successful in properly accounting for errors in the data and in providing unbiased distance estimates. The approximately Bayesian analysis also provides effective model selection on the radial velocity curve.

Based on a representative sample of five stars, our new analyses support the distance scale of Gieren, Barnes, & Moffett (1993, Ap. J., 418, 135) and do not show bias in the calculation of those distances suggested by Laney & Stobie (1995, MNRAS, 274, 337).

# SURFACE BRIGHTNESS METHOD

A synopsis of the technique follows; see Gieren *et al.* (1993) for a full discussion.

- Infer the varying angular diameter $\phi(t)$ from the surface brightness equations and $V_o$, $(V\text{-}R)_o$

  $F_V(t) = A + B\,(V\text{-}R)_o$, where A, B are slight functions of period, and

  $F_V(t) = 4.2206 - 0.1\,V_o - 0.5 \log \phi(t)$, definition of $F_V$

- Infer the linear displacement $\Delta R(t)$ of the stellar atmosphere from the radial velocity curve, $V_r(t)$

  $\Delta R(t) = p\,(V_r(t) - V_\gamma)\,dt$, where $V_\gamma$ is the center of mass $V_r$ and p is the correction from radial velocity to pulsational velocity

- Fit $\phi(t)$ to $\Delta R(t)$ to obtain the mean radius <R> and the stellar distance d

  $\phi(t) = 2\,(\Delta R(t)/d + \langle R \rangle / d)$, where the factor 2 converts angular radius to angular diameter

# LIMITATIONS IN PREVIOUS WORK

- Because both parameters in the fit $\phi(t)$ vs. $\Delta R(t)$ have error in them, the fitting process must properly account for errors or risk a bias in the results. This is an *errors–in–variables problem*.

  – Gieren et al. (1993) ignored this risk and used a linear least squares calculation. Laney & Stobie (1995) correctly criticized their results on this basis and advocated a maximum likelihood solution.

- The radial velocity data must be modeled before integration. This creates a *model selection problem*.

  – Both Gieren *et al.* and Laney & Stobie model the curve $R(t)$ in an *ad hoc* manner, using a hand–drawn mean velocity curves or a Fourier series approximation without an objective choice of the number of terms to include, respectively.

- The error in $\Delta R(t)$ must be properly treated in the solution or the uncertainties in d & $\langle R \rangle$ may be underestimated.

  – Neither Gieren *et al.* nor Laney & Stobie properly account for this in their solutions.

- Our approximately Bayesian method correctly treats all three of these limitations.

# APPROXIMATELY BAYESIAN APPROACH

- We adopt a maximum likelihood approach, which can be considered an approximately Bayesian maximum *a posteriori* estimator with a flat prior. We used the software package GaussFit (Jefferys, Fitzpatrick & McArthur 1988, Cel. Mech., 41, 39). GaussFit solves the *errors–in–variables problem* exactly in a maximum likelihood solution. This addresses the first & third limitations in previous work.

- We solve the model selection problem using the posterior probability computed according to a suggestion by Gull (1988, in *Maximum Entropy and Bayesian Methods in Science and Engineering*, eds. G. J. Erickson & C. R. Smith, p. 153). This permitted an objective decision on how many terms to use in the Fourier Series fit to $V_r(t)$. This addresses the second limitation in previous work.

# DATA SET

- We computed distances and radii for 5 Cepheids:

| Cepheid | Period (days) | E(B-V) (mag) | p ($V_r \rightarrow V_{pul}$) |
|---|---|---|---|
| SZ Tau | 3.14 | 0.294 | 1.375 |
| T Vul | 4.43 | 0.064 | 1.370 |
| U Sgr | 6.74 | 0.403 | 1.365 |
| RY Sco | 20.3 | 0.777 | 1.350 |
| T Mon | 27.0 | 0.209 | 1.347 |

- We adopted exactly the parameters and data used by Gieren *et al.* for a clear comparison of the mathematical methods
  - same photometry, reddening, mean velocity curves, p factor, and parameters (A,B) in the surface brightness equations.

# CALCULATIONS

- We re-determined d and <R> using the same code as Gieren *et al.*
  - We adopted their value for the optimum phase shift between the photometry and solved for d, <R> & $V_\gamma$.

- We then used GaussFit and objective model selection to obtain approximately Bayesian results.
  - We solved for d, <R>, $V_\gamma$ & the optimal phase shift.

# COMPARISON OF RESULTS

| Cepheid | Distance<br>Gieren et al.<br>This paper<br>(parsecs) | Radius<br>Gieren et al.<br>This paper<br>(solar units) | $V_\gamma$<br>(km/s) | Δphase |
|---|---|---|---|---|
| SZ Tau | 649±54 | 41±3 | -3.78 | -.04 |
| | 646±40 | 41±3 | -3.77 | -.034 |
| T Vul | 622±36 | 38±2 | -1.64 | -.04 |
| | 627±34 | 39±2 | -1.63 | -.063 |
| U Sgr | 763±110 | 60±9 | 4.24 | -.04 |
| | 765± 82 | 61±7 | 4.24 | -.047 |
| RY Sco | 1212±84 | 102±7 | -17.68 | -.01 |
| | 1216±52 | 102±4 | -17.67 | +.017 |
| T Mon | 1918±101 | 185±10 | 28.75 | -.01 |
| | 1903±211 | 187±20 | 28.75 | +.006 |

# DISCUSSION

- Use of a rigorous mathematical method to solve the surface brightness (B-W) equations does not change

  – the distances

  – the radii

  – the mean velocities, nor

  – the optimal phase shifts

  obtained from the simple method used in Gieren *et al.* (1993).

- The conjecture by Laney & Stobie (1995) that the mathematical method used by Gieren et al. is biased is shown to be false.

# MORE RESULTS

- Demonstration of the power of Bayesian model selection.
  - We adopted the best available radial velocities for our sample of 5 Cepheids and repeated the GaussFit calculations.
  - Typical results are those of T Mon

    >CORAVEL velocities from

    (45 $V_r$, Bersier *et al.* 1994, A&AS, 108, 25)

| Fourier order (N) | Posterior probability | distance (parsecs) | radius (solar) |
|---|---|---|---|
| 1 | 1.2 E-34 | 1374±316 | 138±31 |
| 2 | 1.6 E-28 | 1576±253 | 158±25 |
| 3 | 7.7 E-26 | 1674±228 | 168±22 |
| 4 | 7.3 E-15 | 1749±158 | 175±15 |
| 5 | 0.943 | 1776±103 | 178±10 |
| 6 | 0.053 | 1857±109 | 186±11 |
| 7 | 0.002 | 1787±131 | 179±13 |
| 8 | 2.9 E-05 | 1809±169 | 181±17 |
| 9 | 1.0 E-06 | 1872±186 | 187±18 |
| 10 | 1.6 E-09 | 2004±227 | 201±22 |
| Probability weighted means = | | 1780±103 | 178±10 |

# MORE DISCUSSION

- Figure 1 - 3: Radial velocity curves for T Mon with N = 4 - 6 models.
  - Our method effectively and objectively selects the optimal number of terms in the Fourier series model of the velocity curve.

- Figure 4 - 5: Distance and radius estimates versus Fourier N for T Mon
  - The distance and radius are not strongly sensitive to the Fourier order selected. (Also true for the optimal phase shift and $V_\gamma$.)